

# **smallantimagmas**

**A library of antiassociative magmas of  
small order**

0.4.1

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# Chapter 1

## smallantimagmas automatic generated documentation

### 1.1 smallantimagmas automatic generated documentation of properties

#### 1.1.1 IsAntiassociative (for IsMagma)

▷ `IsAntiassociative( $M$ )` (property)  
**Returns:** true or false  
identifies whether magma  $M$  is antiassociative.

Example

```
gap> IsAntiassociative(OneSmallGroup(16));  
false  
gap> IsAntiassociative(OneSmallAntimagma(2));  
true  
gap> IsAntiassociative(OneSmallAntimagma(3));  
true
```

#### 1.1.2 IsLeftCyclic (for IsMagma)

▷ `IsLeftCyclic( $M$ )` (property)  
**Returns:** true or false  
if magma is left cyclic  $m$ .

#### 1.1.3 IsRightCyclic (for IsMagma)

▷ `IsRightCyclic( $M$ )` (property)  
**Returns:** true or false  
if magma is left cyclic  $m$ .

#### 1.1.4 IsLeftDistributive (for IsMagma)

▷ `IsLeftDistributive( $M$ )` (property)  
**Returns:** true or false  
if magma is left distributive  $m$ .

Example

```
gap> List(AllSmallAntimagnas(3), M -> IsLeftDistributive(M) );
[ true, false, false, false, false, false, false, false, false, true ]
```

### 1.1.5 IsRightDistributive (for IsMagma)

▷ IsRightDistributive( $M$ ) (property)

**Returns:** true or false  
if magma is right distributive  $m$ .

Example

```
gap> List(AllSmallAntimagnas(3), M -> IsRightDistributive(M) );
[ false, false, false, false, true, false, false, false, true, false ]
```

### 1.1.6 IsLeftCancellative (for IsMagma)

▷ IsLeftCancellative( $M$ ) (property)

**Returns:** true or false  
if magma is left cancellative  $m$ .

Example

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> Display( MultiplicationTable(M) );
[ [ 2, 1 ],
  [ 2, 1 ] ]
gap> IsRightCancellative(M);
false
gap> IsLeftCancellative(M);
true
gap> List(AllSmallAntimagnas(2), M -> IsLeftCancellative(M));
[ true, false ]
```

### 1.1.7 IsRightCancellative (for IsMagma)

▷ IsRightCancellative( $M$ ) (property)

**Returns:** true or false  
if magma is right cancellative  $m$ .

Example

```
gap> List(AllSmallAntimagnas(2), M -> IsRightCancellative(M));
[ false, true ]
```

### 1.1.8 IsCancellative (for IsMagma)

▷ IsCancellative( $M$ ) (property)

**Returns:** true or false  
if magma is cancellative  $m$ .

Example

```
gap> List(AllSmallAntimagnas(2), M -> IsCancellative(M));
[ false, false ]
```

### 1.1.9 IsLeftFPFInducted (for IsMagma)

▷ IsLeftFPFInducted( $M$ ) (property)

**Returns:** true or false

is a left-hand sided fixed-point free inducted  $m$ .

Example

```
gap> Display( MultiplicationTable( SmallAntimagma(2, 2) ) );
[ [ 2, 2 ],
  [ 1, 1 ] ]
gap> IsLeftFPFInducted( SmallAntimagma(2, 2) );
true
```

### 1.1.10 IsRightFPFInducted (for IsMagma)

▷ IsRightFPFInducted( $M$ ) (property)

**Returns:** true or false

is a right-hand sided fixed-point free inducted  $m$ .

Example

```
gap> Display( MultiplicationTable( SmallAntimagma(2, 1) ) );
[ [ 2, 1 ],
  [ 2, 1 ] ]
gap> IsRightFPFInducted( SmallAntimagma(2, 1) );
true
```

### 1.1.11 IsLeftDerangementInducted (for IsMagma)

▷ IsLeftDerangementInducted( $M$ ) (property)

**Returns:** true or false

is a left-hand sided derangement inducted  $m$ .

Example

```
gap> M := SmallAntimagma(2, 2);
<magma with 2 generators>
gap> IsLeftFPFInducted(M);
true
gap> IsRightFPFInducted(M);
false
gap> IsRightDerangementInducted(M);
false
```

### 1.1.12 IsRightDerangementInducted (for IsMagma)

▷ IsRightDerangementInducted( $M$ ) (property)

**Returns:** true or false

is a right-hand sided derangement inducted  $m$ .

Example

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> IsLeftFPFInducted(M);
false
```

```
gap> IsRightFPFInducted(M);
true
gap> IsRightDerangementInducted(M);
true
```

### 1.1.13 IsLeftAlternative (for IsMagma)

- ▷ IsLeftAlternative( $M$ ) (property)  
**Returns:** true or false  
 is a left-alternative magma  $M$ .

Example

### 1.1.14 IsRightAlternative (for IsMagma)

- ▷ IsRightAlternative( $M$ ) (property)  
**Returns:** true or false  
 is a right-alternative magma  $M$ .

Example

## 1.2 smallantimagmas automatic generated documentation of attributes

### 1.2.1 AssociativityIndex (for IsMagma)

- ▷ AssociativityIndex( $M$ ) (attribute)  
 identifies associativity index of  $M$ .

Example

```
gap> OneSmallAntimagma(2);
<magma with 2 generators>
gap> AssociativityIndex(OneSmallAntimagma(2));
0
gap> OneSmallGroup(4);
<pc group of size 4 with 2 generators>
gap> AssociativityIndex(OneSmallGroup(4));
64
gap> AssociativityIndex(OneSmallGroup(4)) = 4 ^ 3;
true
```

### 1.2.2 DiagonalOfMultiplicationTable (for IsMagma)

- ▷ DiagonalOfMultiplicationTable( $M$ ) (attribute)  
 computes diaognal of multiplication table of  $M$ .

Example

```
gap> List(AllSmallAntimagmas(3), M -> DiagonalOfMultiplicationTable((M)));
[ [ 2, 1, 1 ], [ 2, 1, 1 ],
  [ 2, 3, 2 ], [ 2, 1, 1 ],
```

```
[ [ 2, 1, 1 ], [ 2, 1, 2 ],
  [ 2, 3, 2 ], [ 2, 1, 2 ],
  [ 2, 3, 1 ], [ 2, 3, 1 ]
]
```

### 1.2.3 CommutativityIndex (for IsMagma)

▷ `CommutativityIndex( $M$ )` (attribute)

identifies commutativity index of  $M$ .

Example

### 1.2.4 AnticommutativityIndex (for IsMagma)

▷ `AnticommutativityIndex( $M$ )` (attribute)

calculates anticommutativity index of  $M$ .

Example

### 1.2.5 SquaresIndex (for IsMagma)

▷ `SquaresIndex( $M$ )` (attribute)

computes squares index of  $M$  so the order of  $\{m^2 | m \in M\}$ .

Example

```
gap> List(AllSmallAntimagmas(2), M -> List(M, m -> m * m) );
[ [ m2, m1 ], [ m2, m1 ] ]
gap> List(AllSmallAntimagmas(2), M -> SquaresIndex(M ));
[ 2, 2 ]
gap> List(AllSmallAntimagmas(3), M -> SquaresIndex(M ));
[ 2, 2, 2, 2, 2, 2, 2, 2, 3, 3 ]
```

### 1.2.6 IdSmallAntimagma (for IsMagma)

▷ `IdSmallAntimagma( $M$ )` (attribute)

identifies class of antiassociative magma  $M$ .

Example

```
gap> IsAntiassociative(OneSmallGroup(16));
false
gap> IsAntiassociative(OneSmallAntimagma(2));
true
gap> IsAntiassociative(OneSmallAntimagma(3));
true
```



### 1.2.7 LeftOrder (for IsExtLElement)

▷ `LeftOrder([m])` (attribute)

returns a left order of element  $m$ .

### 1.2.8 RightOrder (for IsExtRElement)

▷ `RightOrder([m])` (attribute)

returns a right order of element  $m$ .

### 1.2.9 LeftOrdersOfElements (for IsMagma)

▷ `LeftOrdersOfElements([m])` (attribute)

returns a left order of element  $m$ .

### 1.2.10 RightOrdersOfElements (for IsMagma)

▷ `RightOrdersOfElements([m])` (attribute)

returns a left order of element  $m$ .

## 1.3 smallantimagmas automatic generated documentation of global functions

### 1.3.1 AllSubmagmas

▷ `AllSubmagmas(M)` (function)

builds a collection of non-isomorphic submagmas of  $M$ .

Example

```
gap> AllSmallAntimagmas(2);
[ <magma with 2 generators>, <magma with 2 generators> ]
gap> List(AllSmallAntimagmas(2), M -> AllSubmagmas(M));
[ [ <magma with 1 generator> ], [ <magma with 1 generator> ] ]
```

### 1.3.2 MagmaIsomorphismInvariantsMatch

▷ `MagmaIsomorphismInvariantsMatch(M)` (function)

computes isomorphism invariants of  $M$ .

### 1.3.3 IsMagmaIsomorphic

▷ IsMagmaIsomorphic( $M$ ,  $N$ ) (function)

identifies whether magmas  $M$ ,  $N$  are isomorphic.

Example

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> N := SmallAntimagma(2, 2);
<magma with 2 generators>
gap> T := MagmaByMultiplicationTable([ [2, 1], [2, 1] ]);
<magma with 2 generators>
gap> IsMagmaIsomorphic(M, M);
true
gap> IsMagmaIsomorphic(M, T);
true
gap> IsMagmaIsomorphic(M, N);
false
```

### 1.3.4 IsMagmaAntiisomorphic

▷ IsMagmaAntiisomorphic( $[M, N]$ ) (function)

identifies whether magmas  $M$ ,  $N$  are antiisomorphic.

Example

```
gap> N := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> N := SmallAntimagma(2, 2);
<magma with 2 generators>
gap> IsMagmaAntiisomorphic(M, M);
false
gap> IsMagmaAntiisomorphic(M, N);
true
gap> IsMagmaAntiisomorphic(M, TransposedMagma(M));
true
```

### 1.3.5 TransposedMagma

▷ TransposedMagma( $[M]$ ) (function)

generates transposed magma  $M$ .

Example

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> IsMagmaAntiisomorphic(M, TransposedMagma(M));
true
gap> IsMagmaIsomorphic(M, TransposedMagma(TransposedMagma(M)));
true
```

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> Display(MultiplicationTable(M));
[ [ 2, 1 ],
  [ 2, 1 ] ]
gap> Display(MultiplicationTable(TransposedMagma(M)));
[ [ 2, 2 ],
  [ 1, 1 ] ]
```

### 1.3.6 LeftPower

▷ LeftPower( $[m, k]$ ) (function)

returns a left  $k$ -power of element  $m$ .

### 1.3.7 RightPower

▷ RightPower( $[m, k]$ ) (function)

returns a right  $k$ -power of element  $m$ .

### 1.3.8 AllSmallAntimagmas

▷ AllSmallAntimagmas( $n$ ) (function)

returns all antiassociative magmas of specified size  $n$  (a number)

Example

```
gap> AllSmallAntimagmas(2);
[ <magma with 2 generators>, <magma with 2 generators> ]
gap> AllSmallAntimagmas(3);
[
  <magma with 3 generators>, <magma with 3 generators>, <magma with 3 generators>,
  <magma with 3 generators>, <magma with 3 generators>, <magma with 3 generators>,
  <magma with 3 generators>, <magma with 3 generators>,
  <magma with 3 generators>, <magma with 3 generators>
]
```

### 1.3.9 NrSmallAntimagmas

▷ NrSmallAntimagmas( $n$ ) (function)

counts number of antiassociative magmas of specified size  $n$  (a number).

Example

```
gap> NrSmallAntimagmas(2);
2
gap> NrSmallAntimagmas(3);
10
gap> NrSmallAntimagmas(4);
17780
```

### 1.3.10 SmallAntimagma

▷ `SmallAntimagma( $n$ ,  $i$ )` (function)

returns antiassociative magma of id  $[n, i]$ .

Example

```
gap> SmallAntimagma(2, 1);
<magma with 2 generators>
gap> SmallAntimagma(4, 5);
<magma with 4 generators>
```

### 1.3.11 OneSmallAntimagma

▷ `OneSmallAntimagma( $n$ )` (function)

returns a random antiassociative magma of size  $n$ .

Example

```
gap> OneSmallAntimagma(2);
<magma with 2 generators>

gap> OneSmallAntimagma(3);
<magma with 3 generators>
```

### 1.3.12 ReallyAllSmallAntimagnas

▷ `ReallyAllSmallAntimagnas( $n$ )` (function)

returns really-all antiassociative magmas, isomorphic, of specified size  $n$  (a number)

Example

```
gap> ReallyAllSmallAntimagnas(2);
[ <magma with 2 generators>, <magma with 2 generators> ]
```

### 1.3.13 ReallyNrSmallAntimagnas

▷ `ReallyNrSmallAntimagnas( $n$ )` (function)

counts number of antiassociative magmas of specified size  $n$  (a number)

Example

```
gap> ReallyNrSmallAntimagnas(3);
52
```

### 1.3.14 AntimagmaGeneratorPossibleDiagonals

▷ `AntimagmaGeneratorPossibleDiagonals( $n$ )` (function)

returns all possible diagonals of multiplication table for  $[n]$ -antimagma.

## Example

```
gap> AntimagmaGeneratorPossibleDiagonals(2);
[ [ 2, 1 ] ]
gap> AntimagmaGeneratorPossibleDiagonals(3);
[
  [ 2, 1, 1 ], [ 2, 1, 2 ], [ 2, 3, 1 ], [ 2, 3, 2 ],
  [ 3, 1, 1 ], [ 3, 1, 2 ], [ 3, 3, 1 ], [ 3, 3, 2 ]
]
```

### 1.3.15 AntimagmaGeneratorFilterNonIsomorphicMagmas

▷ AntimagmaGeneratorFilterNonIsomorphicMagmas( $Ms$ ) (function)

filters non-isomorphic magmas  $m$ .

## 1.4 smallantimagmas automatic generated documentation of methods

### 1.4.1 MagmaIsomorphism (for IsMagma, IsMagma)

▷ MagmaIsomorphism( $M$ ,  $N$ ) (operation)

computes an isomorphism between magmas  $M$ ,  $N$ .

## Example

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> N := MagmaByMultiplicationTable([ [2, 1], [2, 1] ]);
<magma with 2 generators>
gap> MagmaIsomorphism(M, N);
<general mapping: Domain([ m1, m2 ]) -> Domain([ m1, m2 ]) >
```

### 1.4.2 MagmaAntiisomorphism (for IsMagma, IsMagma)

▷ MagmaAntiisomorphism( $M$ ,  $N$ ) (operation)

creates an antiisomorphism between magmas  $M$ ,  $N$ .

## Example

```
gap> M := SmallAntimagma(2, 1);
<magma with 2 generators>
gap> N := SmallAntimagma(2, 2);
<magma with 2 generators>
gap> MagmaAntiisomorphism(M, N);
<mapping: Domain([ m1, m2 ]) -> Domain([ m1, m2 ]) >
```

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